Preface to the Special Issue of Jordan Curve Theorem

The Jordan Curve Theorem is an intuitively clear theorem that in rough terms states, "A closed curve divides a plane into the inside and outside areas".

Camille Jordan stated in his publications in 1887 (Cours D'Analysis, Paris) that "this theorem is clearly true", but it was later discovered that the proof of the theorem is in fact quite difficult.

Oswald Veblen insisted that he proved the theorem in 1905 (Theory on Plane Curves in non-Metrical Analysis Situs, Trans. Am. Math. Soc. 6), but the proof relies on intuition in a number of places in the assumed knowledge so it cannot be called a complete proof.

In many areas of mathematics and science (theory of complex functions, electromagnetics, the four color problem, etc.), the Jordan Curve Theorem is assumed inexplicitly and sometimes explicitly.

Since then, many books and papers have been published concerning the Jordan Curve Theorem, but in many cases they introduce references to other books and papers and in tracking down these references, they in turn introduce yet further references and at some point the deepest link of references is lost.

It was clear from the start that the only way to construct a complete proof of the theorem was to build the proofs of all of the necessary mathematics from the foundation level without skipping any steps based on intuition. It was at this time of progress of computers in the background in which the development of proof checker systems began to appear.   In these proof checker systems, computers check mathematical proofs written by humans in a specified grammar.

Since computers must follow strict rules, skipping steps in a proof based on the reasoning that they are intuitive is not tolerated.

Using MIZAR, which is the most popular of the proof checkers, in proving the Jordan Curve Theorem in its entirety has the following kinds of meanings.

1. The fact that the level of mathematics of the Mizar library is not an experimental level, that it is capable of withstanding a sufficiently expansive range of utility can be shown.   Only until recently, such work depended on the paper and the pencil, but with the development of proof checker systems like Mizar, the road to introducing completely new methods in the techniques of mathematics which were not possible is now opened.   As mathematics serves increasingly important roles in various sciences, this is a significant breakthrough for all sciences.

2. This work demonstrates that even theorems which many people and even several scholars think to be trivially clear, to tell the truth, can be extraordinarily difficult and could require several hundreds of thousands of lines to prove.   It is a solid reminder to admonish the overestimation of human intellectual ability.

Based on the points described above, Nakamura advocated proving the Jordan Curve

Theorem in Mizar as the joint project of Japan and Poland.

The time was 1991. Prof. Agata Darmochwał of the Białystok branch of Warsaw University first visited Shinshu University and wrote the article called TOPREAL1 with Nakamura. This opportunity provided the catalyst for more than 10 mathematicians from Japan and Poland to join the project of dividing into pieces and constructing the entire proof. The paper that provided the direction of the proof was written by Yukio Takeuchi former professor of Shinshu University (this proof still contained many portions of intuition which needed to be resolved, but the intuitive portions were fewer in number than in other proofs).

Finally, in September 2005, the final piece of the proof was completed by Dr. Artur Korniłowicz and the project which spanned 14 years was brought to a close on a successful note.

By compiling the proofs of all the divided pieces done by the team members, the complete assembled proof turned out to require more than 200,000 lines. As well as demonstrating the difficulty and involvement of complete logical reasoning, this work also shows the potential of Mizar by accomplishing a major piece of mathematical work. As a result of the culmination of this complete proof of the Jordan Curve Theorem, Mizar will be sure to gain recognition around the world as a new method for mathematics and reasoning.

Here I proposed the special issue of the Jordan Curve Theorem in this journal, which will become a monument of our project and a guidance to all people who wish to know the complete proof of the theorem.

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